

Quantum Discord and Quantum Entanglement in the Background of an Asymptotically Flat Static Black Holes

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Abstract

The quantum discord and tripartite entanglement are discussed in the presence of an asymptotically flat static black holes. The total correlation, quantum discord and classical correlation exhibit decreasing behavior with increasing Hawking temperature. It is shown that the classical correlation is less than the quantum discord in the full range of Hawking temperature. The tripartite entanglements for Greenberger-Horne-Zeilinger and W-states also exhibit decreasing behavior with increasing Hawking temperature. At the infinite limit of Hawking temperature the tripartite entanglements for Greenberger-Horne-Zeilinger and W-states reduce to 52% and 33% of the corresponding values in the flat space limit, respectively.

I. INTRODUCTION

Recently much attention is paid to the quantum information theories in the relativistic framework[1–19]. The most remarkable fact in the inertial frames is the fact that entanglement of given multipartite quantum state is conserved although the entanglement between some degrees of freedom can be transferred to others[4–7]. In non-inertial frames, however, the entanglement is in general degraded, which implies that the quantum correlation between rest and accelerating observers is reduced more and more with increasing the acceleration[8]. The main reason for the reduction of the quantum correlation is that the accelerating observer located in one Rindler wedge loses an information arising from the other Rindler wedge due to the causally disconnected nature between the wedges. This means that some quantum information is leaked into other causally disconnected Rindler space, which makes the reduction of the quantum correlation. In fact, this is a main scenario of the well-known Unruh effect[20, 21]. Recently, this Unruh-type decoherence effect beyond the single-mode approximation is discussed in the context of the quantum information theories[22].

More recently, the quantum entanglement in the black hole background is examined[23, 24]. Especially, in Ref.[23] the Hawking temperature-dependence of the bipartite entanglement is studied in the arbitrary spherically symmetric and asymptotically flat black hole background. The purpose of this paper is to explore the quantum discord[25, 26] and the tripartite entanglement in the same black hole background.

II. SPACETIME BACKGROUND

Throughout this paper we use $G = c = \hbar = k_B = 1$. The metric we consider in this paper is

$$ds^2 = f(r)dt^2 - \frac{1}{h(r)}dr^2 - R^2(r) (d\theta^2 + \sin^2\theta d\varphi^2), \quad (1)$$

where the functions $f(r)$, $h(r)$, and $R(r)$ satisfy $f(\infty) = h(\infty) = 1$, $R(\infty) = r$, and $f(r_H) = h(r_H) = 0$. Thus, the line element (1) includes the various black holes such as Schwarzschild and Reissner-Nordström black holes. The Hawking temperature in this metric is $T_H = \kappa/2\pi$, where κ is a surface gravity defined as $\kappa = \sqrt{f'(r_H)h'(r_H)}/2$.

As shown in Ref. [23] one can consider three-different vacuum states $|0\rangle_{in}$, $|0\rangle_{out}$, and $|0\rangle_K$ in this background. First two vacuum states are the Fock vacua inside and outside

horizon, respectively, and the last one is the Kruskal vacuum outside the event horizon. The interrelation between these vacua is

$$|0\rangle_K = \sqrt{1 - e^{-\omega/T_H}} \sum_{n=0}^{\infty} e^{-n\omega/2T_H} |n\rangle_{in} \otimes |n\rangle_{out}, \quad (2)$$

where $|n\rangle_{in}$ and $|n\rangle_{out}$ are n -particle states constructed from $|0\rangle_{in}$ and $|0\rangle_{out}$ by operating the corresponding creation operators n times, and ω is a frequency of the scalar field. Applying the creation operator of the Kruskal spacetime in Eq. (2) and using the Bogoliubov coefficients, one can construct $|1\rangle_K$, whose expression is

$$|1\rangle_K = (1 - e^{-\omega/T_H}) \sum_{n=0}^{\infty} \sqrt{n+1} e^{-n\omega/2T_H} |n\rangle_{in} \otimes |n+1\rangle_{out}. \quad (3)$$

III. QUANTUM DISCORD

Quantum discord[25, 26] is a measure for *quantumness* of given bipartite quantum state. Usually these two parties consist of system and corresponding apparatus. In this paper, however, we will call these parties as Alice and Bob. We will examine in this section how the quantum discord is changed in the presence of the black hole (1).

We assume that initially Alice and Bob share a entangled state

$$|\psi\rangle_{AB} = \frac{1}{\sqrt{2}} \left(|1\rangle_A |0\rangle_B + |0\rangle_A |1\rangle_B \right) \quad (4)$$

in the asymptotic region. After sharing, Bob moves to the near-horizon region with his own particle detector while Alice stays in the asymptotic region. Since, then, the Bob's detector registers only thermally excited particles due to the Hawking effect, Bob's state can be represented by tensor product of the *in*- and *out*- states. However, since the inside region of the black hole is causally disconnected from Alice and Bob, we have to take a partial trace over *in*-state. Then, the state between Alice and Bob becomes a mixed state whose density matrix becomes

$$\rho_{AB} = \frac{1}{2} |0\rangle_A \langle 0| \otimes M_{00} + \frac{1}{2} |1\rangle_A \langle 1| \otimes M_{11} + \frac{1}{2} |0\rangle_A \langle 1| \otimes M_{01} + \frac{1}{2} |1\rangle_A \langle 0| \otimes M_{10} \quad (5)$$

where

$$\begin{aligned}
M_{00} &= (1 - e^{-\omega/T_H}) \sum_{n=0}^{\infty} e^{-n\omega/T_H} |n\rangle\langle n| \\
M_{11} &= (1 - e^{-\omega/T_H})^2 \sum_{n=0}^{\infty} (n+1) e^{-n\omega/T_H} |n+1\rangle\langle n+1| \\
M_{01} &= (1 - e^{-\omega/T_H})^{3/2} \sum_{n=0}^{\infty} \sqrt{n+1} e^{-n\omega/T_H} |n\rangle\langle n+1| \\
M_{10} &= (1 - e^{-\omega/T_H})^{3/2} \sum_{n=0}^{\infty} \sqrt{n+1} e^{-n\omega/T_H} |n+1\rangle\langle n|.
\end{aligned} \tag{6}$$

It is worthwhile noting $\text{Tr}_B M_{00} = \text{Tr}_B M_{11} = 1$ and $\text{Tr}_B M_{01} = \text{Tr}_B M_{10} = 0$.

Now, we discuss the quantum discord. We assume that Alice performs a projective measurement with a complete set of the measurement operators $\{\Pi_j^A\}$. The usual mutual information between Alice and Bob is

$$I(A : B) = S(A) + S(B) - S(A, B) \tag{7}$$

where S denotes the von Neumann entropy $S(\rho) = -\text{Tr}(\rho \log \rho)$. In our paper, all logarithms are taken to base 2. The classical analogue of Eq. (7) is $I_{cl}(A : B) = H(A) + H(B) - H(A, B)$, where H denotes the Shannon entropy. In classical information theories different representation of the mutual information is $I_{cl}(A : B) = H(A) - H(A|B) = H(B) - H(B|A)$, where $H(X|Y)$ is the conditional entropy of X given Y . The quantum analogue of this representation[25] is

$$J(A : B)_{\{\Pi_j^A\}} = S(B) - \sum_j p_j S(B|\Pi_j^A), \tag{8}$$

where $\{\Pi_j^A\}$ denotes a complete set of the measurement operators prepared by the party A and $S(B|\Pi_j^A)$ is a von Neumann entropy of the party B after the party A has a measurement outcome j . Of course, p_j is a probability for getting outcome j in the quantum measurement. Usual quantum mechanical postulates[27] imply

$$p_j = \text{Tr}_{A,B}(\Pi_j^A \rho_{AB} \Pi_j^A) \quad S(B|\Pi_j^A) = S\left(\rho(B|\Pi_j^A)\right) \tag{9}$$

where $\rho(B|\Pi_j^A) = \text{Tr}_A(\Pi_j^A \rho_{AB} \Pi_j^A)/p_j$. Unlike $I(A : B)$, therefore, $J(A : B)$ is dependent on the complete set of the measurement operators. The quantum discord is defined as

$$\mathcal{D}(A : B) = \min [I(A : B) - J(A : B)] = \min \left[S(A) - S(A, B) + \sum_j p_j S(B|\Pi_j^A) \right], \tag{10}$$

where minimum is taken over all possible complete set of the measurement operators¹.

Now, we would like to compute the quantum discord in the black hole background. From Eq. (5) it is easy to show that $\rho_A \equiv \text{Tr}_B \rho_{AB}$ is a completely mixed state and

$$S(A) = 1. \quad (11)$$

Also it is easy to show

$$S(A, B) = - \sum_{n=0}^{\infty} \Lambda_n \log \Lambda_n \quad (12)$$

$$\Lambda_n = \frac{1}{2} e^{-n\omega/T_H} (1 - e^{-\omega/T_H}) \left[1 + (n+1) (1 - e^{-\omega/T_H}) \right].$$

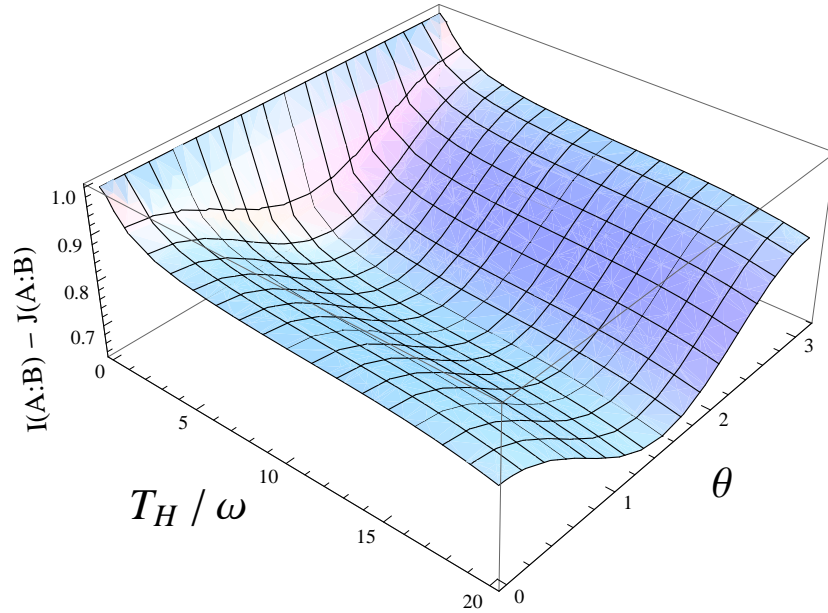


FIG. 1: The θ - and Hawking temperature-dependence of $I(A : B) - J(A : B)$. Minimum of $I(A : B) - J(A : B)$ occurs at $\theta = \pi/2$ in the full range of Hawking temperature.

Now, we introduce the complete set of the projective measurement operators $\{\Pi_1^A, \Pi_2^A\}$ with

$$\Pi_1^A = \frac{I_2 + \mathbf{x} \cdot \boldsymbol{\sigma}}{2} \quad \Pi_2^A = \frac{I_2 - \mathbf{x} \cdot \boldsymbol{\sigma}}{2}. \quad (13)$$

¹ Although authors in Ref. [25] considers the projective measurement, authors in Ref. [26] considers the general measurement including POVM. Thus, the latter is the lower bound of the former.

In Eq. (13) σ denotes the Pauli matrix and $x_1^2 + x_2^2 + x_3^2 = 1$. Then, it is straightforward to show $p_1 = p_2 = 1/2$ and

$$\begin{aligned}\rho(B|\Pi_1^A) &= \frac{1}{2} \left[(1+x_3)M_{00} + (1-x_3)M_{11} + (x_1+ix_2)M_{01} + (x_1-ix_2)M_{10} \right] \\ \rho(B|\Pi_2^A) &= \frac{1}{2} \left[(1-x_3)M_{00} + (1+x_3)M_{11} - (x_1+ix_2)M_{01} - (x_1-ix_2)M_{10} \right].\end{aligned}\quad (14)$$

Since it is impossible to compute the eigenvalues of the $\rho(B|\Pi_j^A)$ ($j = 1, 2$), we should compute $S\left(\rho(B|\Pi_j^A)\right)$ numerically. One can perform this numerical calculation with parametrizing $x_1 = \sin\theta \cos\phi$, $x_2 = \sin\theta \sin\phi$ and $x_3 = \cos\theta$. Then, it is possible to show that the eigenvalues of $\rho(B|\Pi_j^A)$ are independent of ϕ .

The $(T_H/\omega, \theta)$ -dependence of $I(A : B) - J(A : B)$ is plotted in Fig. 1. As this figure exhibits, the minimum is occurred at $\theta = \pi/2$. Therefore, the quantum discord $\mathcal{D}(A : B)$ is obtained from $I(A : B) - J(A : B)$ by letting $\theta = \pi/2$. If we assume that the total correlation is a mutual information $I(A : B)$, it is possible to compute the classical correlation $\mathcal{C}(A : B)$ by

$$\mathcal{C}(A : B) = I(A : B) - \mathcal{D}(A : B). \quad (15)$$

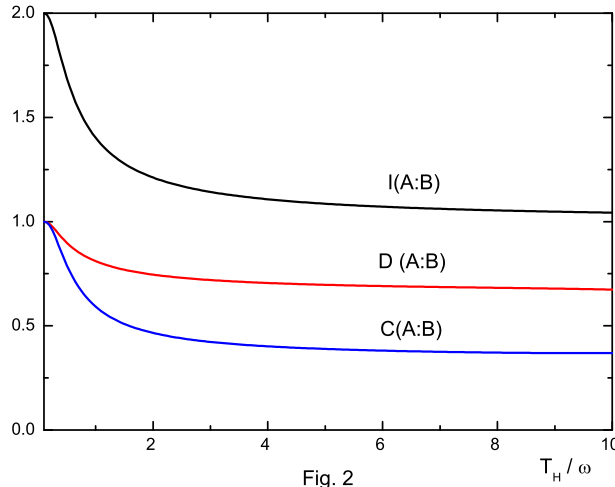


Fig. 2

FIG. 2: The Hawking temperature-dependence of total correlation, quantum discord, and classical correlation. All correlations show a decreasing behavior with increasing the temperature and reduce to 50%, 60%, and 40% of the corresponding values in the flat space limit at $T_H = \infty$.

In Fig. 2 we plot the Hawking temperature-dependence of the total correlation, quantum discord, and classical correlation. As Fig. 2 shows, all correlations exhibit a decreasing behavior with increasing T_H . In the $T_H \rightarrow 0$ limit all correlations approach to the values in the absence of the black hole. In the opposite limit, i.e. $T_H \rightarrow \infty$, $I(A : B)$, $\mathcal{D}(A : B)$, and $\mathcal{C}(A : B)$ approach to 1.0, 0.6, and 0.4, respectively. The remarkable fact is that the classical correlation is less than the quantum discord in the full range of Hawking temperature. Similar behavior was derived when the classical correlation and quantum discord sharing of Dirac field are discussed in the non-inertial frame[12]. In next section we will discuss on the tripartite entanglement in the presence of the black hole (1).

IV. TRIPARTITE ENTANGLEMENT DEGRADATION

The most well-known measure for the tripartite entanglement is a three-tangle[28]. Since, however, the three-tangle is not defined in the qudit system, we cannot use it because of Eq. (2) and Eq. (3). Thus, instead of the three-tangle, we will use π -tangle[29] in this paper as a measure of the tripartite entanglement.

A. Greenberger-Horne-Zeilinger state

Let Alice, Bob, and Charlie share the Greenberger-Horne-Zeilinger (GHZ) state

$$|GHZ\rangle_{ABC} = \frac{1}{\sqrt{2}} [|000\rangle + |111\rangle]_{ABC} \quad (16)$$

in the asymptotic flat region. If Charlie moves to the near-horizon region with his own particle detector, Eq. (2) and Eq. (3) with tracing over the Charlie's *in*-state imply

$$\begin{aligned} & |GHZ\rangle_{ABC} \\ \rightarrow \rho_{ABC} = & \frac{1}{2} \sum_{n=0}^{\infty} e^{-n\omega/T_H} \left[\nu |00n\rangle \langle 00n| + \nu^2 (n+1) |11(n+1)\rangle \langle 11(n+1)| \right. \\ & \left. + \nu^{3/2} \sqrt{n+1} \left\{ |00n\rangle \langle 11(n+1)| + |11(n+1)\rangle \langle 00n| \right\} \right], \end{aligned} \quad (17)$$

where $\nu = 1 - e^{-\omega/T_H}$. Since the Charlie's *out*-state is a qudit state, it is impossible to compute the genuine tripartite entanglement measure called the three-tangle[28]. As we

commented before, therefore, we choose the π -tangle[29] as a tripartite measure defined as

$$\pi = \frac{1}{3}(\pi_A + \pi_B + \pi_C) \quad (18)$$

due to more tractable computation. In Eq. (18) π_A , π_B , and π_C are defined by

$$\pi_A = \mathcal{N}_{A(BC)}^2 - \mathcal{N}_{AB}^2 - \mathcal{N}_{AC}^2 \quad \pi_B = \mathcal{N}_{B(AC)}^2 - \mathcal{N}_{AB}^2 - \mathcal{N}_{BC}^2 \quad \pi_C = \mathcal{N}_{C(AB)}^2 - \mathcal{N}_{AC}^2 - \mathcal{N}_{BC}^2, \quad (19)$$

where $\mathcal{N}_{\alpha(\beta\gamma)} = \|\rho_{ABC}^{T_\alpha}\| - 1$ and $\mathcal{N}_{\alpha\beta} = \|(\text{Tr}_\gamma \rho_{ABC}^{T_\alpha})\| - 1$ with T_α being a partial transposition over α -state and $\|A\| = \text{Tr} \sqrt{AA^\dagger}$. It is easy to show $\pi_{GHZ} = 1$ in the absence of the black hole background.

Now, let us compute the one-tangle $\mathcal{N}_{A(BC)}$. From Eq. (17) it is easy to show that $(\rho_{ABC}^{T_A}) (\rho_{ABC}^{T_A})^\dagger$ is a diagonal. Therefore, the eigenvalues of $(\rho_{ABC}^{T_A}) (\rho_{ABC}^{T_A})^\dagger$ can be computed easily. Since $\|\rho_{ABC}^{T_A}\|$ is a sum of square root of the eigenvalues, one can derive $\mathcal{N}_{A(BC)}$, whose final expression is

$$\mathcal{N}_{A(BC)} = \nu^{3/2} e^{\omega/T_H} Li_{-1/2}(e^{-\omega/T_H}), \quad (20)$$

where $Li_n(z)$ is a polylogarithm function defined as

$$Li_n(z) \equiv \sum_{k=1}^{\infty} \frac{z^k}{k^n} = \frac{z}{1^n} + \frac{z^2}{2^n} + \frac{z^3}{3^n} + \dots \quad (21)$$

Using a property of the polylogarithm function one can show that $\mathcal{N}_{A(BC)}$ approaches to $\sqrt{\pi}/2$ when $T_H \rightarrow \infty$. From a symmetry of the GHZ state it is also easy to show $\mathcal{N}_{B(AC)} = \mathcal{N}_{A(BC)}$.

Now, let us compute the last one-tangle $\mathcal{N}_{C(AB)}$. Since $(\rho_{ABC}^{T_C}) (\rho_{ABC}^{T_C})^\dagger$ becomes

$$(\rho_{ABC}^{T_C}) (\rho_{ABC}^{T_C})^\dagger = D + F, \quad (22)$$

where D and F are

$$\begin{aligned} D &= \frac{1}{4} \sum_{n=0}^{\infty} e^{-2n\omega/T_H} \left[\nu^2 |00n\rangle \langle 00n| + \nu^4 (n+1)^2 |11(n+1)\rangle \langle 11(n+1)| \right. \\ &\quad \left. + \nu^3 (n+1) \left\{ |00(n+1)\rangle \langle 00(n+1)| + |11n\rangle \langle 11n| \right\} \right] \\ F &= \frac{1}{4} \sum_{n=0}^{\infty} e^{-(2n+1)\omega/T_H} \left[\nu^{5/2} \sqrt{n+1} \left\{ |11n\rangle \langle 00(n+1)| + |00(n+1)\rangle \langle 11n| \right\} \right. \\ &\quad \left. + \nu^{7/2} (n+1) \sqrt{n+2} \left\{ |11(n+1)\rangle \langle 00(n+2)| + |00(n+2)\rangle \langle 11(n+1)| \right\} \right], \end{aligned} \quad (23)$$

the off-diagonal part F makes it difficult to compute the eigenvalues of $(\rho_{ABC}^{T_C})(\rho_{ABC}^{T_C})^\dagger$. However, one can make $(\rho_{ABC}^{T_C})(\rho_{ABC}^{T_C})^\dagger$ block-diagonal by ordering the basis as $\{|000\rangle, |110\rangle, |001\rangle, |111\rangle, |002\rangle, |112\rangle, \dots\}$. Thus, one can compute the eigenvalues of $(\rho_{ABC}^{T_C})(\rho_{ABC}^{T_C})^\dagger$ analytically, which are $\left\{ \nu^2/4, \Lambda_n^\pm \middle|_{n=0,1,2,\dots} \right\}$. Here, Λ_n^\pm are eigenvalues of each block given by

$$\Lambda_n^\pm = \frac{\nu^2}{8} e^{-2n\omega/T_H} \left[(\mu_n^2 + 2\nu) \pm \mu_n \sqrt{\mu_n^2 + 4\nu} \right], \quad (24)$$

where $\mu_n = n e^{\omega/T_H} \nu + e^{-\omega/T_H}$. Therefore, $\mathcal{N}_{C(AB)}$ reduces to

$$\mathcal{N}_{C(AB)} = \frac{\nu}{2} + \sum_{n=0}^{\infty} \left(\sqrt{\Lambda_n^+} + \sqrt{\Lambda_n^-} \right) - 1. \quad (25)$$

Finally, one can show that all two tangles \mathcal{N}_{AB} , \mathcal{N}_{AC} , and \mathcal{N}_{BC} are identically zero.

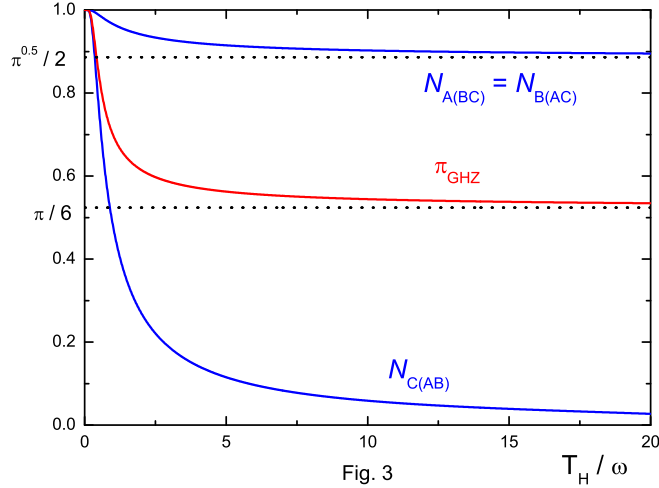


Fig. 3

FIG. 3: The Hawking temperature-dependence of one tangles and π_{GHZ} . The π -tangle decreases with increasing T_H , and eventually reduces to $\pi/6 \sim 0.524$ at $T_H = \infty$.

The one-tangles and π -tangle are plotted in Fig. 3 as a function of Hawking temperature. As this figure shows, the π -tangle decreases with increasing the Hawking temperature, and eventually reduces to $\pi/6 \sim 0.524$ at $T_H \rightarrow \infty$. At $T_H = 0$ the π -tangle exactly coincides with that in the absence of the black hole. Thus, as expected, the tripartite entanglement is degraded when Charlie moves to the near-horizon region from the asymptotic region with his own particle detector.

B. W state

Let Alice, Bob, and Charlie share the W-state[30]

$$|W\rangle_{ABC} = \frac{1}{\sqrt{3}} [|001\rangle + |010\rangle + |100\rangle]_{ABC} \quad (26)$$

in the asymptotic flat region. It is easy to show that the π -tangle for the W-state is $\pi_W = 4(\sqrt{5} - 1)/9 \sim 0.55$ in the flat space limit.

By following similar calculation to the case of GHZ state, one can compute the Hawking temperature-dependence of π_W in the presence of the black hole background. We do not want to repeat the computational procedure again in this paper. Instead, we present Fig. 4, which shows one-tangles, two-tangles, and π_W as a function of Hawking temperature. In Fig. 4 (a) we plot the one- and two-tangles. All tangles exhibit decreasing behavior with increasing Hawking temperature except \mathcal{N}_{AB} , which is independent of T_H . At $T_H \rightarrow \infty$ $\mathcal{N}_{A(BC)}$ and $\mathcal{N}_{B(AC)}$ approaches to 0.659 while $\mathcal{N}_{C(AB)}$ has a vanishing limit. The remarkable fact is that the two-tangles \mathcal{N}_{AC} and \mathcal{N}_{BC} becomes abruptly zero in the region $T_H > 1.45\omega$. This reminds us of the concurrence, one of the bipartite entanglement measure. In Fig. 4 (b) we plot π_W as a function of T_H . At $T_H = 0$ π_W in the flat space is recovered. However, it exhibits a decreasing behavior with increasing T_H , and eventually reduces to 0.18 at $T_H = \infty$ limit.

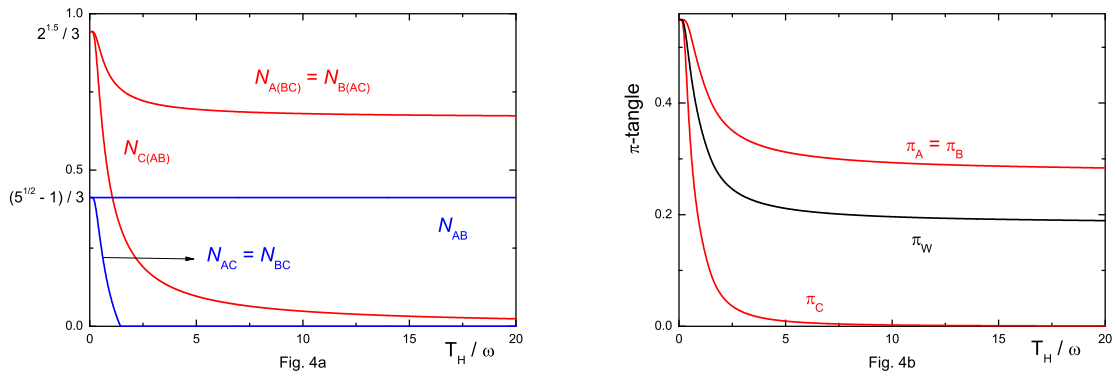


FIG. 4: (a) The Hawking temperature-dependence of one- and two-tangles. (b) The Hawking temperature-dependence of π_W . As a case of GHZ state π_W decreases with increasing the temperature, and eventually reduces to 0.18 at $T_H = \infty$.

V. CONCLUSION

In this paper we discussed the quantum discord and tripartite entanglement in the presence of the asymptotically flat static black holes. Both the quantum discord and the tripartite entanglement exhibit decreasing behavior with increasing Hawking temperature. This implies that the presence of the black holes reduces the quantum correlation when one party moves from asymptotic to near-horizon regions with his (or her) own particle detector.

Although we have not commented here, the tripartite entanglement of Alice, Bob, and Charlie's *in* state (or AntiCharlie) does not completely vanish. Probably, this fact implies that some quantum information processes can be performed partially across the black hole horizon. To confirm this it seems to be important to compute the teleportation fidelity by making use of the tripartite teleportation scheme[31, 32]. If the tripartite teleportation is possible, even if incompletely, across the horizon, what this means in the context of causality? The answer may be important in the context of quantum gravity. We would like to explore this issue in the future.

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